A duplicate standard asset allocation funds Methods: A tool for reducing long-run risk

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It seems difficult to build models that represent reality. Financial-risk models, in particular, get investors in trouble. A model is a theoretical construct which used some unrealistic hypotheses. Since, many investors suffer now from the mis-pricing investment risk. The aim objective of this article was to test the efficiency of a strategy, incorporating some options and seeking to super-duplicate the realizations of a standard-asset allocation policy. The replication strategy allows reducing imperfections effects, and by consequence long-run risk. The replication means optimizing two objective-functions: MSE (Mean-squared Errors) and WMSE (Weighted Mean-squared Errors).

Numerical tests on replication efficiency concern a long-run investment with over the country (OTC) financial options. Results proved the presence of a Buy-and-Hold portfolio based on options that is more efficient than an active trading strategy. The optimal behaviour of the economic agent is a function of the number, the type, as well as its position on the options. To fix his decision, he can negotiate with his bank one of the strategies maximizing his preferences, independently of the imperfections of the market.

Key words: Long-run risk, replication principal, standard allocation policy, portfolio management, transaction costs, derivatives market.

INTRODUCTION

A fundamental question in the field of finance is how investors can reduce risk and achieve their goals. For volatile rates of returns, the dynamic investment seems expensive or in defect underestimated (Henrotte, 1993; Grannan and Swindle, 1996; Toft, 1996; Martellini, 2000; Brennan and Xia, 2001). Although, Buy and Hold strategy allows reducing the effects of imperfections, it however, increases the loss of capital. The current study therefore, looks for an outstanding strategy to minimize the deviation from theoretically-derived optimal asset allocation.

Gary (1992) finds bonds more volatile than stocks and proves the performance of the Buy and Hold strategy for an investment in Dow Jones Index from 1946 to 1991. This result is sustained by Dudenhausen (2002) who hold that Buy and Hold strategy always works with long-term investors. Conversely, other studies show that trading investors preferred hedging strategy and selected less risk investment perspectives. As Mulvey and Simsek (2001) note, investors with long horizon always prefer to gain more income, since they placed large amount of their funds in risky investment. And for Campbell (2002), cash is more volatile than bonds and stocks. This result questions the efficiency of the static approach and argues that the behaviour of a short trading investor is different from long trading investor.

For a long-run investor case, we developed and tested two trading strategies. The first is a standard asset allocation as developed by Merton (1969), and without any imperfections, such as bankruptcy costs and transaction costs, the effect of taxation, etc. While the second blends both a Buy-and-Hold strategy and financial options.

In conformity with Merton (1995), we discovered that the introduction of the options improves and stabilizes certain policies in particular assets optimal allocation.

Introducing the options on the portfolio has some applications in actuarial science. For instance, Carr and Madan (2000) consider the optimal allocation problem; where investor is permitted to trade stocks and
derivatives, and when the underlying price process follows a pure jump Levy. Carr et al. (2001) develop a variant of the single-period model where investors allocate their wealth between stock, bond and European options with a range of strike prices. Liu and Pan (2003) examine the efficiency of options in a dynamic investment policy with event-risk.

In this article, we examined the following proposition: under certain characteristics of options, investors can define a Buy-and-Hold strategy that duplicates expectations of a standard-asset allocation policy. We followed a logic inspired from the model of Black and Scholes (1973): that a dynamic portfolio value can duplicate an instantaneous price of a European call option. We retained two objective-functions to illustrate the principal of replication. The first objective-function allows optimizing Mean-squared Errors (MSE) by involving errors-square minimization among wealth expectations related to standard and passive asset strategies. The parameter Root Mean-squared Errors (RMSE) measures the replication cost or error. The second objective-function is to minimize mean-squared errors between end-period wealth expectations, weighted by investor’s tolerance (WMSE). The current research focused on evaluating the performance of the investment portfolios on both Weighted Root Mean-squared Errors (WWRMSE) and Equivalent certainly measures.

In opposition to several researches that support the actively-managed portfolio effectiveness, the finding results prove that a Buy-and-Hold portfolio consisting of just a fewer options is an excellent strategy to reduce long-run risk.

In the following, we developed, in section 2, the standard model of Merton (1969, 1971). Then, we present the Buy-and-Hold strategies. Section 3, examines analytically the effectiveness of these duplicated strategies. Conclusion and final comments are presented in the last section.

**METHODOLOGY**

**A standard-asset allocation problem**

We study Merton’s standard allocation problem for an investor whose preferences are defined by a power-utility function:

\[ U(W_T) = \frac{W_T^\gamma}{\gamma} \]  

(1)

Where, \( \gamma \) is the degree of risk-aversion, strictly inferior to one.

The investor is allowed to maximize his expected utility function defined on terminal wealth \( W_T \) as follows:

\[ \max_{\omega} \mathbb{E}[U(W_T)] \]  

(2)

The investor can trade in a risk-free bond and stock over a finite investment horizon \([0,T]\). The bond yields an instantaneous return of \( \delta dt \) and an initial market price of Euro 1, the bond price at any time “t” is simply exp \( (\delta t) \). The stock price is denoted by \( S(t) \), and is typically assumed to satisfy the following Itô stochastic differential equation:

\[ dS_t = \mu S_t dt + \sigma S_t dW_t \]  

(3)

\( B_t \) is a one-dimensional standard Brownian motion defined on a complete probability space \( (\xi, F, P) \) with a natural filtration \( F = \{ F_t, 0 < t < T \} \). The mean \( \mu \) and standard deviation \( \sigma \) are constant.

Based on these assumptions as well as the self-financing hypothesis, the dynamics of wealth process for \( 0 < t < T \) can be written as following:

\[ dW(t) = (\delta + \omega(t) (\mu - \gamma) W(t) dt + \omega(t) W(t) \sigma dW_t \]  

(4)

With \( W(0) \) is the initial wealth at time 0.

The investor attempts to find the fraction of wealth \( \omega(1) \) invested in the stock. For Merton (1969, 1971), the standard allocation means holding along the time the following ratio of stock-bond:

\[ \omega^* = \frac{\mu - \delta}{\sigma^2 (1 - \gamma)} \]  

(5)

Let \( \pi = \frac{\mu - \delta}{\sigma} \), and we assume that there are no transaction costs in the market. The terminal wealth expectation is given by:

\[ W^*(T) = W(0) \exp \left( \delta T - \frac{\pi^2 T (2 \gamma - 1)}{2(1 - \gamma)^2} + \frac{\pi B_T}{(1 - \gamma)} \right) \]  

(6)

The close-form solution considers the case of an investor who would like to allocate an initial wealth of Euro 100 000 for ten years, and his risk tolerance is defined by a constant risk-aversion coefficient of “-1”. At an initial time, the stock and bond prices are \( S(0)=50 \) and \( S_0(0)=1 \), respectively. In addition, \( \mu=0.1 \) and \( \sigma=0.25 \) are appropriate to the geometric Brownian motion parameters. Then, the standard-asset allocation provides a fraction of wealth on stock of 41 percent, according to a Merton ratio (5) of 0.7. Once this ratio holds along the investment horizon, the investor can understand an expected end-period wealth (6) of Euro 200 800.

**Replication strategy problems**

The principle of this research seems simple. We seek to build a portfolio that can reproduce or replicate Merton expectations. For the performance measure, we used cumulative deviation between standard allocation and Buy and Hold realizations. A portfolio is optimal when its expected terminal wealth closed to the expected terminal wealth of the standard allocation.

Two functions illustrate the principal of replication. The first consists in optimizing Mean-squared Errors
(MSE), portfolios are driven from the minimization of errors among terminal wealth expectations: \( W^*_T \text{ and } V^*_T \), correspond respectively to standard and Buy-and-Hold strategies. The evaluation of the optimal portfolio focused on RMSE measure that allows reporting the square root of errors to the terminal wealth expectation resulting from the standard-asset allocation. The investor selects portfolios with the smallest RMSE, and the best replication corresponds to RMSE equal to zero:

\[
\text{Min } \mathbb{E}\left[ (W^*_T - V^*_T)^2 \right] (7)
\]

RMSE (\%) = \[ \frac{1}{W^*_T} \sqrt{\sum_i (W^*_T - V^*_T)^2} \]

Note that Merton’s problem acts only on the weight of stock to hold, while the mean-square error strategy involves the monetary quantities of stock (a), bond (b), and put (p).

Several arguments can justify the choice of a MSE objective-function: First, the MSE strategy involves an expected terminal wealth that maximizes investor’s preferences with perfectly-efficient markets assumption. Second, the absence of preferences in MSE permits an investment policy as a function of market parameters and independent of the behaviour of investors. Third, the MSE approach can lead to other problems, which do not retain the maximization of utility as an objective-function (e.g. dollar cost averaging).

The second objective-function consists in minimizing the errors among terminal wealth expectations weight by the degree of risk-aversion. This function shows hybrid by introducing the tolerance of investor against standard weight deviations, in objective-function. Then, the WMRSE takes into account the limits of the MSE approach:

\[
\text{Min } \mathbb{E}\left[ U^- (W^*_T - V^*_T)^2 \right] (8)
\]

The evaluation measures are according to the WMRSE as to the certainly equivalent \((CE_3)\). The last measure reveals investor’s indifference between the standard-asset allocation and the weighted mean-squared-asset allocation: if \((CE_3)\) close to \((CE_0)\), the investor will be indifferent:

\[
\mathbb{E}(V^*_T) = U(CE_R)
\]

To prevent theoretically-derived asset allocation, we articulated on the principle of replication when implementing a Buy-and-Hold strategy. This last strategy is an interesting one for several reasons. In fact, for Bertsimas, Kogan and Lo (2000): it is currently impossible to trade continuously, and even if it was possible, market frictions would render continuous trading too costly. Campbell and Vicier (2002), and others, show that a Buy-and-Hold strategy is more suited to long-term investment horizon, since it avoids the necessity of frequent portfolio revision. Gary (1992) proves empirically that a Dow Jones index, as a passive management strategy, is more profitable than active strategies.

Based on passive strategy implementation, the budgetary equations (4) can be converted into the following linear equations:

\[
V_t = aS_0(0)\exp(rT) + bS_p(T) + \sum_i c_i D_{ii} + \sum_j p_j D_{2j} (9)
\]

\[
W(0) = \exp(-rT)E^0(V_T) (10)
\]

The operator \( E^0 [ ] \) represents the conditional expectation of the terminal wealth \( V_T \) in \( Q \) equivalent to \( P \): \( P \) is the space of probability relative to the standard-asset allocation. The calculation of the current value of assets requires the knowledge of \( Q \), and not that of \( P \). We adopted the similar \( Q \) relative to the neutral risk space, as supposed to be in option-pricing theory. Therefore, the stock terminal value is as the following Martingale equation:

\[
\tilde{S}_j(t) = S_j(0)\exp(\sigma W_t - \sigma^2 t/2) \text{With}
\]

\[
W_t = B_t + \frac{\mu - r}{\sigma} t (11)
\]

For each sub-problem, the simulation of equation (10) means taking a set of 4000, where each point has a probability 1/4000. The study focused also on Black and Scholes (1973) to evaluate options written only on stock:

- **Call option**
- **Put option**

\[
D_{1j} = (P_j - K_j)^+ = \text{Max}(0, P_j - K_j) \quad D_{2j} = (E_j - P_j)^+ = \text{Max}(0, E_j - P_j)
\]

\( E, K \) : settlement prices of call and put respectively.

Because a long-run option is not yet present in organization markets, the study assumed a trimester-binomially tree to simulate the option’s strike price. This simulation generates 41 strike prices: \( n=41 \). Among the range of strike prices, the optimization employs only three in maximum. Moreover, in order to ensure a non-negative wealth the optimization must respect the following solvency constraints:

<table>
<thead>
<tr>
<th>A. Call option</th>
<th>B. Put option</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ a exp (rT)</td>
<td>0 ≤ a exp (rT)</td>
</tr>
<tr>
<td>0 ≤ a exp (rT) + b K</td>
<td>0 ≤ a exp (rT) + b K</td>
</tr>
<tr>
<td>0 ≤ a exp (rT) + b K</td>
<td>0 ≤ a exp (rT) + b K</td>
</tr>
<tr>
<td>0 ≤ a exp (rT) + (b + c) K</td>
<td>0 ≤ a exp (rT) + (b + c) K</td>
</tr>
<tr>
<td>0 ≤ a exp (rT) + (b + c) K</td>
<td>0 ≤ a exp (rT) + (b + c) K</td>
</tr>
</tbody>
</table>

According to past assumptions, the Buy-and Hold strategy may correspond to a straightforward nonlinear optimization problem, with a concave objective-function and no-linear constraints. The problem may be consistent to Kuhn-Tucker (KT) solution.

\[ ^{1} \text{Cf. Muthuraman and Kumar (2004).} \]

\[ ^{2} \text{In first time, the KT optimization requires an examination of optima for every type of solution determined by a premium order condition. In second time, the KT Algorithm proceeds comparing all local solutions,} \]
RESULTS AND DISCUSSION

Table 1 contains mean-squared portfolios containing between one to three options; each one is selected from a range of 41 strike prices. Particularly, three added-option strategies lead to the Buy-and-Hold investigation: (i) call, (ii) put and (iii) straddle. Table 1 presents only portfolios with a replication cost close to zero. The top panel corresponds to the case of adding a call strategy. The analysis of portfolios confirms that if the number of call increases, the RMSE decreases, but the expected end-period utility does not increase regularly.

In addition, Table 1 describes monetary quantities for each opportunity chosen. The analysis emphasizes significant dissimilarity of asset shares between the standard-asset allocation and the mean-squared portfolios. In MSE optimization, the initial wealth is considerably concentrated on bond, while the investment on stock appears to be weak. Thus, the asset shares fluctuate with the number of options added. In the case of three calls, the optimal Buy-and-Hold portfolio involves putting wealth on a variety of calls. Though based on RMSE, the super-replication makes up of shorting only two calls. Given a current stock price of Euro 50, calls chosen are deeply out-of-the-money; their prices are extremely close to zero and taking into account the end-period price expectation (Euro 136), strike prices are still high.

The middle panel of Table 1 presents the replication results with adding one to three puts. The excellent opportunity emphasizes long in-the-money puts. The hold quantities of at-the-money and out-of-the-money puts are weak. The comparison between the two high panels of Table 1 reveals no quite difference of stock and bond positions. However, the amount allowed to stock is important, even if the replication centres on straddle strategy. The bottom panel of Table 1 details this strategy, and reveals the opportunity to sell, simultaneously, calls and puts for outperforming Buy-and-Hold optimization. The options chosen are on the whole in-the-money.

It follows from this premium-finding results that options characteristics are very fuzzy. However, investing on a unique option is sufficient for constructing an efficient Buy-and-Hold portfolio. Liu and Pan (2003) prove that the optimal number of options is function of the number of price jumps. Carr et al., (2001) suggest that two puts are sufficient to outperform wealth allocation in a dynamic setting. The current article falls in line with this suggestion: with just weak number of options, the investor can apply an efficient mean-squared portfolio.

Portfolios summarized in Table 2 derive from the WMSE-objective function. The top panel exposes the efficient portfolios including only calls. In contrast with Table 1, there is a weak investment on bonds. The investor allows all his original wealth and the premium acquired from shorting calls, on stock. The analysis of asset positions reveals insights into a correspondence of the MSE strategy and the hedge theory: the investor takes a long position on stocks and a short one on puts. In other terms, the investor profits from option-insurance to take some risky positions on stock. Considering the put case, the middle panel of Table 2 shows that if the number of puts increases, the WRMSE decreases and tends gradually to zero. The super-duplicate strategy takes out in a portfolio with three puts. This portfolio corresponds also to an important certainty equivalent measurement. Therefore, a perfect homogeneity occurs between both accepted measures for the replication evaluation.

Characteristics of options appear to be different for each optimization opportunity. A comparison between Tables 1 and 2 highlights this fact. In spite of all differences, portfolios reported in these tables are efficient. They do confirm the main idea of this article: the options are an excellent instrument of investment (Merton, 1995).

We noticed that the unique problem related to implying a replication portfolio is at the outset of the investment horizon. For instance, Table 2 proves the existence of many efficiency portfolios. Among these portfolios, the super-replication consists in adding three puts options defined by small strike prices. The fact to conclude an option with strike price of Euro 5 seems to be very theoretical, but it is possible in OTC market. At each optimization, MSE and WMSE, the investor can define a set of Buy-and-Hold portfolios that allow expectation no longer different from the standard-asset allocation policy.

It seems to be important that an investor keeps up a comparison between all expectations derived separately from actively and duplicated strategies before taking any decision. After the negotiation with his banker about options, the investor can fix the strategy maximizing his preferences.

For several reasons, the results of this paper have some limits. In fact, the analysis does not support the variation of some economic determinants: the degree of aversion of risk and the volatility of securities. In addition, the different results were derived essentially from the standard model of Merton (1969). However, the financial literature proposes more other sophisticated models. Furthermore, this study considers only the integration of a European option. Finally, the procedures developed may be useful in considering more general version of a portfolio problem such as the inclusion of intermediate consumption. Another interesting extension would be the case of two or more risky assets. These issues can be discussed in further research.

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3 A large number of published works deals with the analysis of hedging portfolio efficiency: e.g., Yates and Kopprash (1980), Merton et al. (1982), Bookstaber and Clarke (1981, 1984), Leland (1985)...

4 Haugh and Lo (2001) solved the problem with Asiatic options.
Table 1. Replication portfolios: Mean-squared error function

<table>
<thead>
<tr>
<th>Strike Prices</th>
<th>Optimal allocation</th>
<th>EU</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond</td>
<td>Stock</td>
<td>Call 1</td>
</tr>
<tr>
<td>E1</td>
<td>224</td>
<td>1.13 e5</td>
<td>124.61</td>
</tr>
<tr>
<td>E1</td>
<td>1294</td>
<td>1.21 e5</td>
<td>9.38 e-6</td>
</tr>
<tr>
<td>E1</td>
<td>1662</td>
<td>1.21 e5</td>
<td>16.04 e-5</td>
</tr>
<tr>
<td>K1</td>
<td>6</td>
<td>1.21 e5</td>
<td>1.4 e-5</td>
</tr>
<tr>
<td>K1</td>
<td>174</td>
<td>1.21 e5</td>
<td>9.42 e-5</td>
</tr>
<tr>
<td>K1</td>
<td>4522</td>
<td>2265.1</td>
<td>43.57</td>
</tr>
<tr>
<td>E1</td>
<td>2134</td>
<td>1.21 e5</td>
<td>1.09 e-5</td>
</tr>
</tbody>
</table>

Note: Risk-aversion coefficient: -1; Investment horizon: 10 years; Interest rate: 5 %; Bond price: Euro 1; Stock price: Euro 50; Standard deviation: 25 %; Average mean: 12%; Initial wealth: Euro 100 000; RMSE: root mean-squared errors; EU: Expected utility; E: Call strike price; K: Put strike price.

Table 2. Replication portfolios: Weighted mean-squared Error function

<table>
<thead>
<tr>
<th>Strike Prices</th>
<th>Optimal allocation</th>
<th>EU</th>
<th>CE&lt;sub&gt;R&lt;/sub&gt;</th>
<th>WRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond</td>
<td>Stock</td>
<td>Call 1</td>
<td>Call 2</td>
</tr>
<tr>
<td>E1</td>
<td>14</td>
<td>24.6</td>
<td>8 153.4</td>
<td>-7 401.1</td>
</tr>
<tr>
<td>E1</td>
<td>30</td>
<td>18</td>
<td>73344</td>
<td>-1 003</td>
</tr>
<tr>
<td>E1</td>
<td>100</td>
<td>8.21 e-11</td>
<td>42 339</td>
<td>-7.38 e8</td>
</tr>
<tr>
<td>E1</td>
<td>584</td>
<td>8.21 e-11</td>
<td>42 339</td>
<td>-7.38 e8</td>
</tr>
<tr>
<td>K1</td>
<td>136</td>
<td>.02</td>
<td>1.2129</td>
<td>.97</td>
</tr>
<tr>
<td>K1</td>
<td>14</td>
<td>72.865</td>
<td>553.1</td>
<td>-7859</td>
</tr>
<tr>
<td>K1</td>
<td>5</td>
<td>1.10 e5</td>
<td>9.58 e-10</td>
<td>-2.85 e7</td>
</tr>
<tr>
<td>E1</td>
<td>5</td>
<td>22.224</td>
<td>18 149</td>
<td>-17 192</td>
</tr>
</tbody>
</table>

Note: Risk-aversion coefficient: -1; Investment horizon: 10 years; Interest rate: 5 %; Bond price: Euro 1; Stock price: Euro 50; Standard deviation: 25 %; Average mean: 12%; Initial wealth: Euro 100 000; WRMSE: Weighted-root mean-squared errors; CE<sub>R</sub>: certainly equivalent of the replication strategy; EU: Expected utility; E: Call strike price; K: Put strike price; Iteration: 1000 times.

Conclusion

This paper examined the case of an investor seeking to optimize his allocation at long-run horizon. The recent advancement in modern portfolio theory emphasizes the effectiveness of the active asset allocation (Goasling, 2000; Dahlquist and Harvey, 2001). In this context, most studies suggest that the actively-managed portfolio corresponds to a hedge portfolio or a discontinuous asset allocation. However, our findings allow the following conclusions: (i) under certain characteristics of options, the active asset allocation is not necessarily profitable than a passive strategy. Maintaining a passive strategy while purchasing some options, the investor can reach a
situation, optimizing his preferences independently of market imperfections. (ii) Options are an excellent instrument of insurance, speculation and investment\(^5\).

Indeed, the economy needs long funds to assure its short and long-term equilibrium. Comparatively to earlier studies, our reflection was concerned about an investor trying to place its wealth over ten years. With costly economic assumption, the investor can build a portfolio based on options, having a performance superior to a Merton policy rules. This insurance against economy imperfections improve long-term investor’s welfare, and stimulate them to become more integrated into the economy.

Finally, a long-term option can encourage investors to apply a long-run investment policy, which can be a factor to improve financial market stability (Merton, 1995). However, several researchers advocate option’s responsibility for the economy destabilization. Therefore, an enormous control of options must be hold in order to prevent a further sub prime crisis in future.

REFERENCES


