Issues in Scientific Research Vol.2 (1),pp. 1-17, February 2018
Available online at http://www.journalissues.org/ISR/
https://doi.org/10.15739/ISR.18.001

ISSN 2408-7513

Original Research Article

Theoretical investigations of two-phase flow and heat transfer in parallel multichannel core of a low power reactor

Received 4 December, 2017 Revised 17 January, 2018 Accepted 21 January, 2018 Published 27 January, 2018

Edward Shitsi[2]*,
Edward HK Akaho[2]
and
Benjamin JB Nyarko[1]

[1]Department of Nuclear Reactors Research Centre, National Nuclear Research Institute, Ghana Atomic Energy Commission, P.O. Box LG 80, Legon-Accra, Ghana.
[2]Graduate School of Nuclear and Allied Sciences, University of Ghana, P.O. Box AE1, Atomic Energy, Ghana.

*Corresponding Author E-mail: edwardshitsi@yahoo.com

Ghana Research Reactor-1 (GHARR-1) is a low power reactor of 30 kW (th) tank-in-pool type with parallel multichannel core, operating presently with a Highly Enriched Uranium (HEU). The data on GHARR-1 was used for this research work. The GHARR-1 under normal operating conditions experiences single-phase flow. Under abnormal operating conditions GHARR-1 experiences two-phase flow regime which is caused by large reactivity insertions associated with the abnormal operating conditions of the low power reactor. This theoretical research purposely investigated two-phase flow properties (parameters) which are normally associated with nucleate boiling that develops at abnormal operating conditions of the low power reactor. A FORTRAN code, conservation laws and correlations were employed to compute the two-phase flow parameters, which include boiling boundary, vapor quality, pressure drop, single-phase and nucleate boiling heat transfer coefficients, onset of nucleate boiling (ONB) heat flux, critical heat flux and safety margins (ONB ratio (ONBR) and DNB ratio (DNBR)). The boiling boundary of 229.2 mm from the inlet at 37 kW reactor power, ONBR of 1.23 and DNBR of 134.44 were obtained for 30 kW GHARR-1. For abnormal reactor powers from 40-260 kW, the values of -0.005-0.106, 2.203-1.153 kPa, 1.161-1.795 kW/m^{2}K, 4.013-14.878 kW/m^{2}K, 11.949-158.730 kW/m^2 and 2951.917-2953.687 kW/m^2 were obtained for exit vapor quality, pressure drop, single-phase and nucleate boiling heat transfer coefficients, ONB heat flux and critical heat flux respectively. These results indicated that subcooled nucleate boiling will commence if the 30 kW GHARR-1 is operated above 37 kW reactor power, the coolant in the GHARR-1 core will not evaporate if it is operated below 40 kW, coolant heat removal from the core at low reactor powers would be more effective and low power reactors operating at low reactor powers can hardly experience nuclear accident. At abnormal operating conditions with high reactor powers, the low power GHARR-1 is safe and can have longer life span if it is continued to be operated at low reactor powers.

Key words: Low power reactor, heat transfer, single-phase flow, two-phase flow, safety margins.

INTRODUCTION

The reactor GHARR-1 is a 30 kW (th) tank-in-pool type, low power research reactor, operating presently with a Highly Enriched Uranium (HEU) core. The GHARR-1, which is a Chinese constructed Miniature Neutron Source Reactor similar to Canadian SLOW POKE, is designed to be compact and inherently safe. The reactor is mainly used for Research

Table 1. Technical specifications of Ghana MNSR (Birikorang et al., 2010).

Parameters	Description
Reactor type	Tank-in-pool
Rated thermal power	30 kW
Fuel	UAl_4 dispersed in Al
U-235 enrichment	90.2%
Core shape	Cylindrical
Core diameter	23.0 cm
Core height	23.0 cm
Number of fuel element	344
Weight of U-235	998.116 g
Number of irradiation channel	10
Inner channel	5
Outer channel	5
Flux in inner channel at full power	1×10^{12} n $cm^{-2}s^{-1}$
Flux in outer channel at full power	5×10^{11} n $cm^{-2}s^{-1}$
Reactor cooling mode	Natural convection
Height of inlet orifice	6 mm
Height of outlet orifice	7 mm
Diameter of fuel meat	4.3 mm
Diameter of fuel element	5.5 mm
Excess core reactivity	3.99 mk
Length of cadmium control rod	230 mm

and Development (R&D) in nuclear technique and nuclear engineering, Neutron Activation Analysis (NAA), production of short-lived radioisotopes, human resource development for Ghana's nuclear power programme and for education and training. The technical specification of Ghana MNSR is shown in Table 1. Figure 1 and Figure 2 show cross sectional view through reactor core and schematic diagram of the coolant flow pattern respectively. A channel is assumed to be an equilateral triangle which is bound by three cylindrical fuel rods of the same size shown in Figure 3. The core is cooled by natural convection. The heat generated by fission in the multichannel is transferred in an upward direction and the hot water, which moves to the upper section of the reactor tank, is transferred to the water in the pool.

For safety assessment of the reactor GHARR-1 and in order to update the Safety Analysis Report, Akaho et al developed a computer code to simulate the peak powers that could be associated with large reactivity insertions of 6.71 mk and 9 mk during the time of abnormal reactor operations such as installation of a new core (fresh fuel replacement) and addition of incorrect thickness of top Be plates in the Al tray located at the top of the reactor core respectively (Akaho and Maaku, 2002). Peak powers of 187.23 kW and 254.30 kW for reactivity insertions of 6.71 mk and 9 mk respectively were predicted. For both cases, it was indicated that large surface heat fluxes would be generated and saturation temperature would be reached, and also subcooled nucleate boiling was expected to occur within the flow channels of the reactor for these reactivity insertions. They also predicted a peak power of 100.8 kW for inserting ramp reactivity of the cold clean core excess reactivity of 4 mk, which compared favorably with the experimental value of 100.2 kW (Akaho and Maaku, 2002). It was also indicated that subcooled nucleate boiling was expected within the flow channels of the reactor for this predicted peak power of 100.8 kW.Ampomah-Amoako et al., 2009 performed thermal-hydraulic analysis on GHARR-1 using Program for the Analysis of Reactor Transients/Argonne National Laboratory (PARET/ANL) code. It was indicated that Ghana Research Reactor-1 is safe to operate under 2.1mk and 4mk insertions of reactivity. Mweetwa et al., 2017 performed similar thermal-hydraulic analysis on GHARR-1 using the same simulation code after nineteen years of operation of the reactor. It was also indicated that the reactor is still safe to operate in the range 2.1 mk to 4.0 mk reactivity insertions. Mangena (2017) performed thermal hydraulic analysis of the Miniature Neutron Source reactor, Ghana Research Reactor-1 (GHARR-1) using STAR-CCM+ CFD code. The reactor has a Highly Enriched Uranium (HEU) core consisting 344 fuel pins arranged in 10 multi-concentric circle layers at a pitch distance of 10.95 mm. The study considered only the first two concentric rings with varying power of 15 to 30kW. The choice of the first two concentric rings was informed by computational resources available and the results of the computation of local power peaking factors based on power densities. The results of the study show that all computed accident situation temperatures were below the melting point of the U-Al alloy satisfying regulatory requirements for safe operations of the GHARR-1 facility. Ahmed et al., 2008 carried an experimental study to investigate the effect

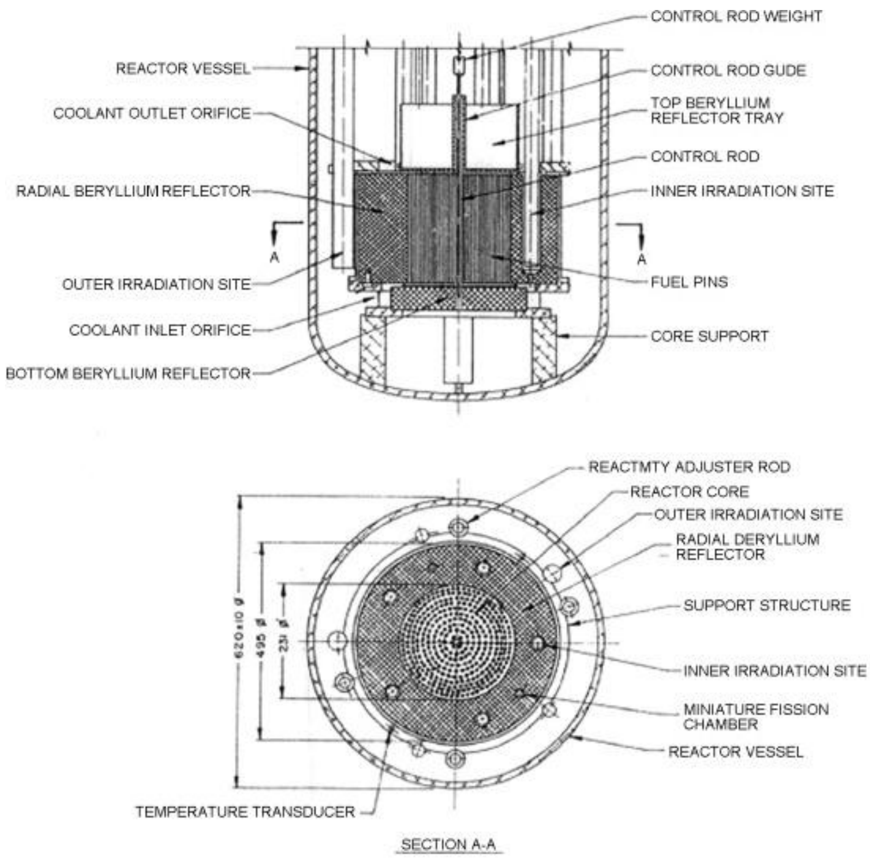

Figure 1: Cross sectional view of GHARR-1 core (Ampomah-Amoako et al., 2009)

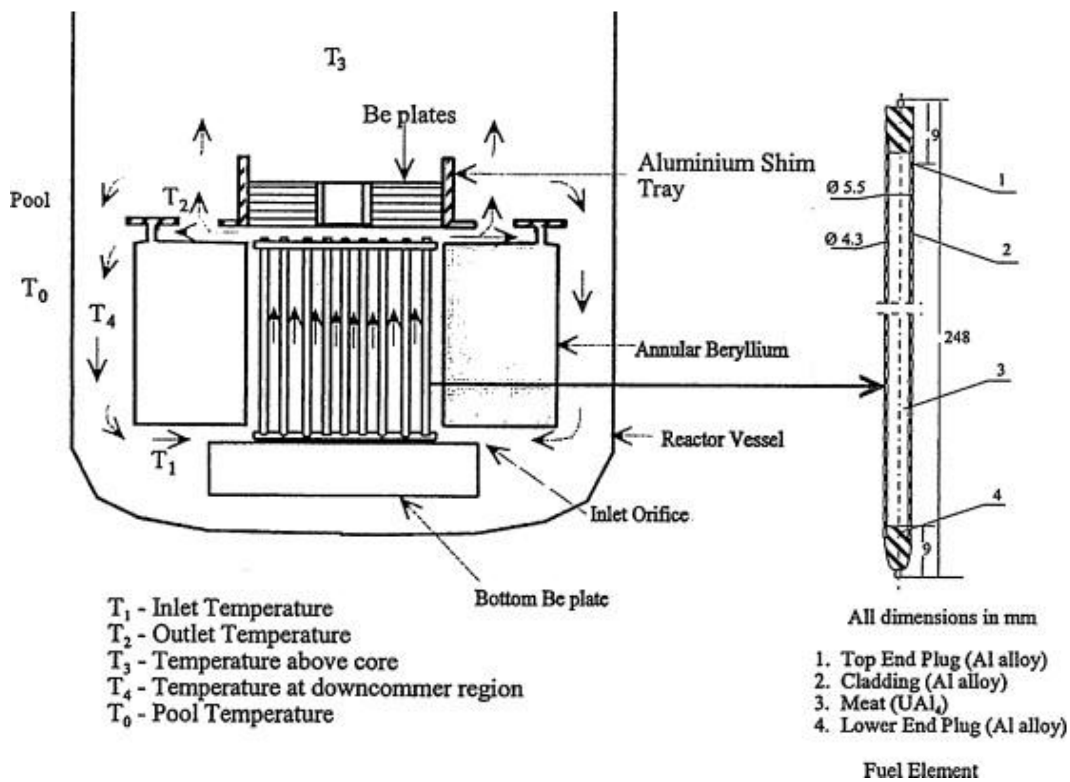

Figure 2: Schematic diagram of the coolant flow pattern (Ampomah-Amoako et al., 2009).

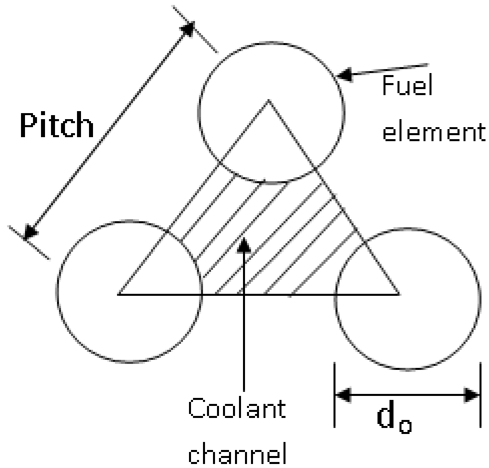

Figure 3: Coolant channel

of coolant temperature on reactor power using Nigeria Research Reactor-1 NINR-1. The experimental data obtained were in good agreement with the data obtained using a semi-empirical relationship between the reactor power, core inlet temperature and the coolant temperature rise. This semi-empirical relationship between reactor power, coolant inlet temperature and coolant temperature rise is used to predict power level of NIRR-1 and other low power research reactors such as GHARR-1. Abdulhameed and Balogun (2017) carried out numerical study using PARET to predict peak temperatures of Nigeria Research Reactor (NIRR-1) core components under several reactivity accident tests. At power levels below 80kW, there were no significant differences between the peak fuel centerline temperature, the peak fuel surface temperature and the peak clad surface temperature in the hot channel as well as in the average channel. The results from the reactivity accident tests show that power can never rise to an uncontrollable level in the core of NIRR-1 under ramp or step insertion of up to 4mk of reactivity. The calculated temperature of the important core components (e.g. fuel and clad) in the two channels (during this reactivity accident test) were far below their melting point temperatures. Boiling of any kind was not observed during this reactivity accident test. It was therefore stated that NIRR-1 could be operated safely even if there is an inadvertent addition of up to 4mk of positive reactivity.

The knowledge of heat transfer phenomenon associated with liquid vapor phase-change in nuclear reactors is of great importance to the nuclear engineer. The GHARR-1, which under normal reactor operating conditions has its flow through the channels as single phase experiences two phase flow regime due to reactor power excursion because of large reactivity insertions associated with abnormal operations of the low power reactor(Akaho and Maaku, 2002). These large reactivity insertions that associate with abnormal operating conditions with high reactor powers could be experienced during the time of abnormal operations such as installation of a new core, addition of incorrect thickness of top Be plates in the Al tray located at the top of the core and insertion of ramp reactivity of the cold clean core. The two phase flow theoretical investigations would enable the estimation of two-phase flow properties which are associated with the abnormal operating conditions of GHARR-1. The two-phase flow properties include boiling boundary between single-phase and two-phase flows, vapor quality, pressure drop, heat transfer coefficients, onset of nucleate boiling heat flux, critical heat flux and safety margins.

For safe operation of the low power reactor GHARR-1 during normal operation and at the time of abnormal operation, there is the need to determine the boiling boundary between the single-phase flow and subcooled nucleate boiling, and other two-phase flow properties that are associated with the subcooled nucleate boiling that develops at those abnormal reactor powers. Hence there is the need for this theoretical research work to quantitatively determine the two-phase flow properties that are associated with the subcooled nucleate boiling at abnormal reactor powers for safe operation of GHARR-1 facility. The safety of operation of nuclear reactors being the main concern of nuclear engineers, this theoretical research work also determined the adequate safety margins that might be allowed to take care of the abnormal reactor operating conditions. Thus the estimation of two-phase flow properties and safety margins is important for safe operation of GHARR-1 facility.The laws of conservation of mass, momentum and energy in one dimension were used over the active region of the reactor core (the fuel element section) to determine the boiling boundary, coolant exit velocity and pressure drop. Appropriate correlations were also used to determine the heat transfer coefficients, heat flux at the onset of nucleate boiling, critical heat flux and safety margins.

NOMENCLATURE

Roman symbols

A_{cv} cross sectional area of the control volume (active core) (m^2)
C_p specific heat ($Jkg^{-1}K^{-1}$)
d_e equivalent diameter (m)
d_o fuel diameter (m)
g gravitational acceleration, 9.8 (ms^{-2})
h enthalpy (J/kg)
h_{fg} latent heat of vaporization (J/kg)
k, k_f thermal conductivity of the fluid (W/m K)
L_H length or height of the active core (m)
L_N boiling boundary from the inlet (m)
m mass flow rate (kg/s)
ttime (s)
T temperature (°C)
t_c, $T_w$clad outer surface or wall temperature (°C)
T_{SAT} coolant saturation temperature (°C)
u coolant velocity (m/s)
V_{cv} volume of the active core (m^3)
v_{fg} difference in specific volume of saturated liquid and vapor (m^3/kg)
x vapor quality

Greek symbols

β expansion coefficient (K^{-1})
α void fraction
Δ difference
ΔP pressure drop (Pa)
ΔT_{SAT} wall superheat, difference between coolant saturation temperature and wall temperature above the saturation temperature (°C), $T_W - T_{SAT}$
$\Delta T_{SAT_{ONB}}$ difference between coolant saturation temperature and wall temperature at the Onset of Nucleate Boiling ONB (°C), $T_{WONB} - T_{SAT}$
$(\Delta T_{SUB})_i$ inlet subcooling degree, difference between coolant temperatures at its saturation and at the inlet of the active core (°C), $(T_f)_i - T_{SAT}$
ρ density (kg/m^3)
1∅ single-phase
2∅ two-phase
$\varphi_{2\emptyset}$ two-phase friction multiplication factor
μdynamic viscosity ($kgm^{-1}s^{-1}$)

Subscripts

avaverage
cvcontrol volume
e, ex exit
ggas or vapor
i inlet
lliquid
LO liquid alone
nb nucleate boiling
onb, ONB onset of nucleate boiling
dnb, DNB departure from nucleate boiling
SAT saturation
SBL subcooled boiling length
SUB subcooling

Dimensionless parameters and definitions

Gr Grashof number, $\dfrac{d_e^{\,3}\rho^2 g\beta\Delta T}{\mu^2}$

Pr Prandth number, $\dfrac{\mu C_p}{k}$

Re Reynolds number, $\dfrac{\rho u d_e}{\mu}$

Abbreviations

CHF critical heat flux
CFD computational fluid dynamics
DNB departure from nucleate boiling
DNBR departure from nucleate boiling ratio
NB nucleate boiling
ONB onset of nucleate boiling
ONBR onset of nucleate boiling ratio

THEORY

The conservation of mass, momentum and energy equations were applied to the active height section of the multichannel core of GHARR-1 at the abnormal reactor power levels. Figure 4 shows the active height section of the core at the abnormal reactor power levels.

The general assumptions are as follows:

1. Constant coolant properties at a given system pressure.
2. Homogeneous flow model for two-phase flow.
3. Uniform heat flux in the axial direction.
4. The fluid enters the channel in subcooled state.
5. The two-phases are in thermodynamic equilibrium.
6. The active height section of the core was considered.

The following conservation of mass, momentum and energy equations in one dimension respectively were employed to calculate the boiling boundary, coolant exit velocity and pressure drop across the active height section of GHARR-1 core at the abnormal reactor power levels associated with abnormal operations (Equations 1-3).

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho u) = 0 \quad (1)$$

$$-\frac{\partial P}{\partial z} = \frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial z}(\rho u^2) + \left[\frac{f}{de} + \frac{\sum_{i=1}^{N} K_i}{\Delta z}\right]\frac{\rho u^2}{2} + \rho g \quad (2)$$

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial z}(\rho h u) = q''' \quad (3)$$

where ρ is density (kg m^{-3}); *t*, time (s); *u*, velocity (m/s); *z*, axial coordinate (m); *f*, friction factor; *de*, equivalent diameter (m); *k*, loss coefficient; *g*, gravity constant (m/s^2); *P*, pressure (Pa); *h*, enthalpy (J/kg); and q‴, power density (W/m^3).

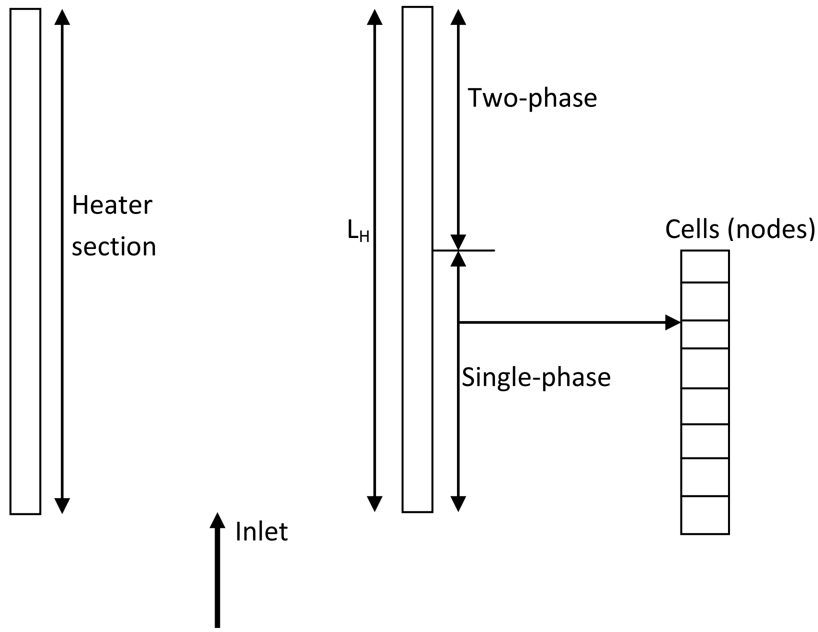

Figure 4: Active height section of GHARR-1 core at abnormal reactor powers

The equivalent diameter of a coolant channel was estimated using the relation given by Equation (4).

$$d_e = \frac{4\left[\frac{1}{2}p^2 sin60^\circ - \frac{1}{2}\left(\frac{\pi}{4}\right) d_o{}^2\right]}{\frac{1}{2}\pi d_o} \quad (4)$$

where$p$ is the pitch of a coolant channel (m) and *do*, the fuel outside diameter (m)

Thermodynamic properties of the coolant at the exit of the heater section at any operating power were determined using the coolant exit temperature. The coolant exit temperature is given by the relation (Akaho et al., 2003) obtained using the thermal-hydraulic test data (Equation 5).

$$T_e = T_f + 6.81P^{(0.59+0.0019T_f)}T_f^{-0.35} \quad (5)$$

where$P$ is the reactor power (kW), and T_f and T_e are respectively inlet and exit temperatures (°C) of the coolant in the core.

Calculation of boiling boundary

Conservation of one dimensional single-phase energy Equation (3) was used to determine the boiling boundary between the single-phase and nucleate boiling regions (Guo et al., 2008). Heat transferred from the active channel length (heater section) to the coolant is proportional to the operating reactor power. The one dimensional single-phase energy equation estimates the active channel length of only the single-phase region. If the active channel length (boiling boundary) estimated using the single-phase energy equation is more than the total active channel length (230 mm), then the operating power used in the energy equation cannot cause nucleate boiling (Guo et al., 2005). If the active channel length estimated is less than the heater section, then the operating power used in the energy equation can cause nucleate boiling. The nucleate boiling length is the difference between the total active channel length and the boiling boundary.

The following assumptions were made over the single phase zone:

1. The water (coolant) density up to the boiling boundary L_N is constant.
2. The single-phase region is subdivided into nodes N, having equal length as shown in Figure 4.
3. The velocity variation of the coolant in the single-phase region is negligible.

The energy equation was integrated over one node with boundaries, L_n and L_{n-1}, to obtain Equation (6).

$$L_n = \frac{\rho h V_{cv} t u_i + L_{n-1}(Pt - \rho h V_{cv})}{Pt - \rho h V_{cv}} \quad (6)$$

whereu_i (u) is the coolant inlet velocity (m/s), h is enthalpy (J/kg) for one node, q''' is power density (W/m^3), energy released per unit volume of the active reactor core, L_N is the boiling boundary (m) from the inlet of the active core, p is power (W), t is time (s), ρ is coolant density (kg/m^3), V_{cv} is the volume of the active reactor core and N is the cell (node) number of the single-phase. The detailed derivation of L_n is shown in the Appendix.

The boiling boundary L_N was used to find the fluid boiling boundary enthalpy h_N by assuming that the enthalpy variation of the bulk fluid through the heated channel is linear. The coolant boiling boundary enthalpy $h_N$at any reactor power is given by Equation 7.

$$h_N = \frac{L_N}{L_H}(h_e - h_i) + h_i \quad (7)$$

where h_e is the coolant exit enthalpy at the exit of the heater section for any reactor power and L_H is the length of the heater section.

The coolant boiling boundary enthalpy h_N is greater than the coolant exit enthalpy h_e if the boiling boundary L_N is greater than the heater section length L_H. This means that the reactor operating power cannot cause nucleate boiling.

Calculation of the exit velocity of the coolant

To calculate the coolant exit velocity, it was assumed that nucleation would not have much significant effect on the coolant velocity in the two-phase region.

Coolant exit velocity was obtained by combining the conservation of one dimensional mass and energy equations (Akaho and Maaku, 2002; Guo et al., 2008; Ampomah-Amoako et al., 2009) (Equation 8).

$$u_e = u_i + \frac{pt - V_{cv}\rho h}{V_{cv}t\rho h}\Delta L_{SBL} \quad (8)$$

where u_i and u_e are the inlet (boiling boundary) and exit coolant velocities respectively,

ρ is the coolant density in the nucleate boiling region, p is power, t is time, ΔL_{SBL} is subcooled boiling length and h is subcooled boiling enthalpy. The detailed derivation of L_N and u_e is shown in the Appendix.

The calculated coolant exit velocity is the coolant exit velocity if the total flow in the two-phase region is purely liquid (single-phase).

The boiling boundary enthalpy h_N (Equation 7) was used to estimate the boiling boundary temperature of the fluid using thermodynamic data from steam tables. The fluid boiling boundary temperature was then used to read the fluid densities.

Pressure drop estimation

To obtain the total pressure across the multichannel, the conservation of one dimensional momentum equation (2) was integrated with respect to the axial distance, from the inlet to exit of the heater section (Guo et al., 2008) (Equation 9).

$$\Delta P_H = \Delta P_I + \Delta P_a + \Delta P_f + \Delta P_g \quad (9)$$

where ΔP_H, ΔP_I, ΔP_a, ΔP_f, and ΔP_g are total, inertial, acceleration, frictional and gravitational pressure drops respectively.

Estimation of inertial pressure drop

Integrating the inertial pressure drop component of Equation (2), Equation (10) was obtained.

$$\Delta P_I = \int_0^{L_H}\left\{\frac{\partial}{\partial t}(\rho u)\right\}dz = (L_H - L_N)\left\{\frac{\rho_{2\emptyset}(u_e - u_i) + u_{av}(\rho_e - \rho_i)}{t}\right\} \quad (10)$$

where $\rho_{2\emptyset}$ is the two-phase density given by Equation (11) and u_{av} is the coolant average velocity in the two-phase region given by Equation (12) (Guo et al., 2008).

$$\rho_{2\emptyset} = \rho_i \frac{ln(\rho_i/\rho_e)}{(\rho_i/\rho_e) - 1} \quad (11)$$

$$u_{av} = \frac{u_i + u_e}{2} \quad (12)$$

The two-phase coolant exit density ρ_e is given by Equation (13) (Guo et al., 2008).

$$\rho_e = \frac{1}{\left[v_f + \frac{v_{fg}}{h_{fg}}(h_e - h_N)\right]} \quad (13)$$

where v_f is the specific volume of saturated liquid (m^3/kg), v_{fg} the difference in specific volume of saturated liquid and vapor (m^3/kg), h_{fg} the latent heat of vaporization (J/kg), ρ_i the boiling boundary density (kg/m^3), h_e and h_N are the coolant enthalpies (J/kg) at the exit and boiling boundary respectively.

Inertial pressure loss was neglected because of the small values of inlet and exit coolant velocities pertaining to the MNSR operating at low power values.

Estimation of acceleration pressure drop

Integrating the acceleration pressure drop component of Equation (2), Equation (14) was obtained.

$$\Delta P_a = \int_0^{L_H}\frac{\partial}{\partial z}(\rho u^2)dz = \rho_e u_e{}^2 - \rho_i u_i{}^2 \quad (14)$$

where ρ_e is two-phase coolant exit density.

Estimation of frictional pressure drop

The frictional pressure drop was estimated by integrating the frictional pressure drop component of Equation (2), which gives Equation (15)

$$\Delta P_f = \int_0^{L_H}\frac{f}{de}\frac{\rho u^2}{2}dz = \frac{1}{2}f_{1\emptyset}\frac{L_N}{de}\rho_{1\emptyset}u_i{}^2 + \left(\frac{1}{2}f_{2\emptyset LO}\frac{(L_H - L_N)}{de}\rho_{LO}u_{av}{}^2\right)\varphi_{2\emptyset} \quad (15)$$

The single-phase friction factor $f_{1\emptyset}$ is given by the relation (Akaho and Maaku, 2002) (Equation 16).

$$f_{1\emptyset} = \frac{0.184}{R_e{}^{0.2}} = 0.184\left(\frac{\mu_{1\emptyset}}{\rho_{1\emptyset}u_i d_e}\right)^{0.2} \quad (16)$$

where $\rho_{1\emptyset}$ and $\mu_{1\emptyset}$ are single-phase coolant density and viscosity respectively.

The friction factor $f_{2\emptyset LO}$ for liquid alone in the two-phase region was estimated using the relation (Akaho and Maaku, 2002) (Equation 17).

$$f_{2\emptyset LO} = \frac{0.184}{R_e{}^{0.2}} = 0.184\left(\frac{\mu_{LO}}{(1 - x_{ex})\rho_{LO}u_{av}d_e}\right)^{0.2} \quad (17)$$

where ρ_{LO} and μ_{LO} are respectively the density and viscosity for the liquid alone in the two-phase region.

The exit thermodynamic equilibrium vapor quality x_{ex} at any power P was estimated using relationship derived based on energy balance on the active reactor core (Akaho and Maaku, 2002) (Equation 18).

$$x_{ex} = \frac{P}{mh_{fg}} - \frac{h_N - h_i}{h_{fg}} \qquad (18)$$

where h_i and h_N are the enthalpies of the coolant at the inlet and boiling boundary respectively (J/kg), *m* is mass flow rate (kg/s) and h_{fg} is the enthalpy of vaporization (J/kg).

The two-phase friction multiplication factor $\varphi_{2\emptyset}$ was estimated using the Levy's method (Guo et al., 2008) (Equation 19).

$$\varphi_{2\emptyset} = \frac{(1 - x_{ex})^{1.75}}{(1 - \alpha)^2} \qquad (19)$$

The void fraction α across the flow channel was estimated using Bankoff variable density model for bubble flow as reported by Hewitt with no local slip between the two phases (Akaho and Maaku, 2002) (Equation 20).

$$\alpha = \frac{K_B \rho_l x_{ex}}{\rho_g - x_{ex}(\rho_g - \rho_l)} \qquad (20)$$

where ρ_g and ρ_l are densities of the gas (vapor) and liquid at the exit of the active core respectively and K_B is a parameter which has a value of 0.89 using experimental data for steam-water systems.

The additional frictional pressure loss due to local loss coefficient (drag coefficient) K_i (the value of $K_{entrance}$ = 0.5 and K_{exit} = 1.0 (Akaho and Dagadu, 1999)) in Equation (2) was neglected because of the small value of the coolant velocity pertaining to the MNSR operating at low power values.

Estimation of gravitational pressure drop

The gravitational component of the pressure drop is obtained from Equation (2) as follows (Equation 21):

$$\Delta P_g = \int_0^{L_H} \rho g \, dz = \rho_{1\emptyset} g L_N + \rho_{2\emptyset} g (L_H - L_N) \qquad (21)$$

Calculation of single-phase heat transfer coefficient

The laminar single-phase heat transfer coefficient *h* was estimated using the relation (Akaho and Maaku, 2002) given by Equation (22).

$$h = \frac{k_f}{d_e} 0.68 (Gr.Pr)^{0.25} \qquad (22)$$

Gr is the Grashof number defined by Equation (23).

$$G_r = \frac{d_e{}^3 \rho^2 g \beta \Delta T}{\mu^2} \qquad (23)$$

Pr is the Prandth number defined by Equation (24).

$$P_r = \frac{\mu C_p}{k} \qquad (24)$$

ΔT (the temperature difference between the wall and the bulk coolant) was estimated by iteration (Adiutori, 2004). First, an initial estimate ΔT_1 was selected and its corresponding h_1 was calculated using the correlation above (Equation 22). ΔT_2 was calculated using the actual heat flux q_a and h_1 ($\Delta T_2 = q_a/h_1$). Iteration was continued until convergence and an estimate value of *h* was obtained.

Calculation of nucleate boiling heat transfer coefficient

Nucleate boiling heat transfer coefficient h_{nb} was estimated using Mikheev correlation (Akaho and Dagadu, 1999) given by Equation (25).

$$h_{nb} = 3.0 q^{0.7} P^{0.15} \qquad (25)$$

where

q is power density in W/m^2 and P is pressure in bar.

Calculation of onset of nucleate boiling heat flux

The Bergles and Rohsenow correlation (Tanaka et al., 2001) was adopted to calculate onset of nucleate boiling heat flux q_{ONB}. The correlation is given by Equation (26).

$$q_{ONB} = 0.0018 P^{1.156} [1.8(T_{WONB} - T_{SAT})]^{2.828/P^{0.0234}} \qquad (26)$$

where P is the pressure with unit MPa, and T_{WONB} and T_{SAT} are the wall temperature at the onset of nucleate boiling and saturation temperature in °C, respectively.

The wall temperature at the onset of nucleate boiling T_{WONB} is given by another form of Bergles and Rohsenow correlation (Garland et al., 1999) as (Equation 27):

$$T_{WONB} = T_{SAT} + \frac{5}{9}\left(\frac{9.23 q''}{P^{1.156}}\right)^{0.463 P^{0.0234}} \qquad (27)$$

where

q'' is the heat flux (W/cm^2) that can cause boiling and P is pressure in bar.

Calculation of critical heat flux

Mirshak correlation (Akaho and Dagadu, 1999) was used to calculate critical heat flux. Mirshak correlation is given by Equation (28).

$$q_{DNB} = 1.51 \times 10^6 [1 + 0.1198u][1 + 0.00914 \Delta T_{SUB}][1 + 0.19P] \qquad (28)$$

ΔT_{SUB} is the inlet subcooling, *u* is the velocity of flow in m/s, P is pressure in bar and q_{DNB} is the heat flux for departure from nucleate boiling DNB in W/m^2.

Safety margins

The boiling boundary analysis was used to obtain the ONBR of the 30 kW MNSR. Bergles and Rohsenow ONB heat flux correlation was also used to observe the trend of variation of ONBR with reactor power.

The DNBR is defined as the ratio of critical heat flux to the local heat flux (actual operating heat flux) (El-Sawy et al., 2001).

Mishark correlation was used to calculate the DNBR of the 30 kW MNSR. The Mishark correlation was also used to observe trend of variation of DNBR with reactor power.

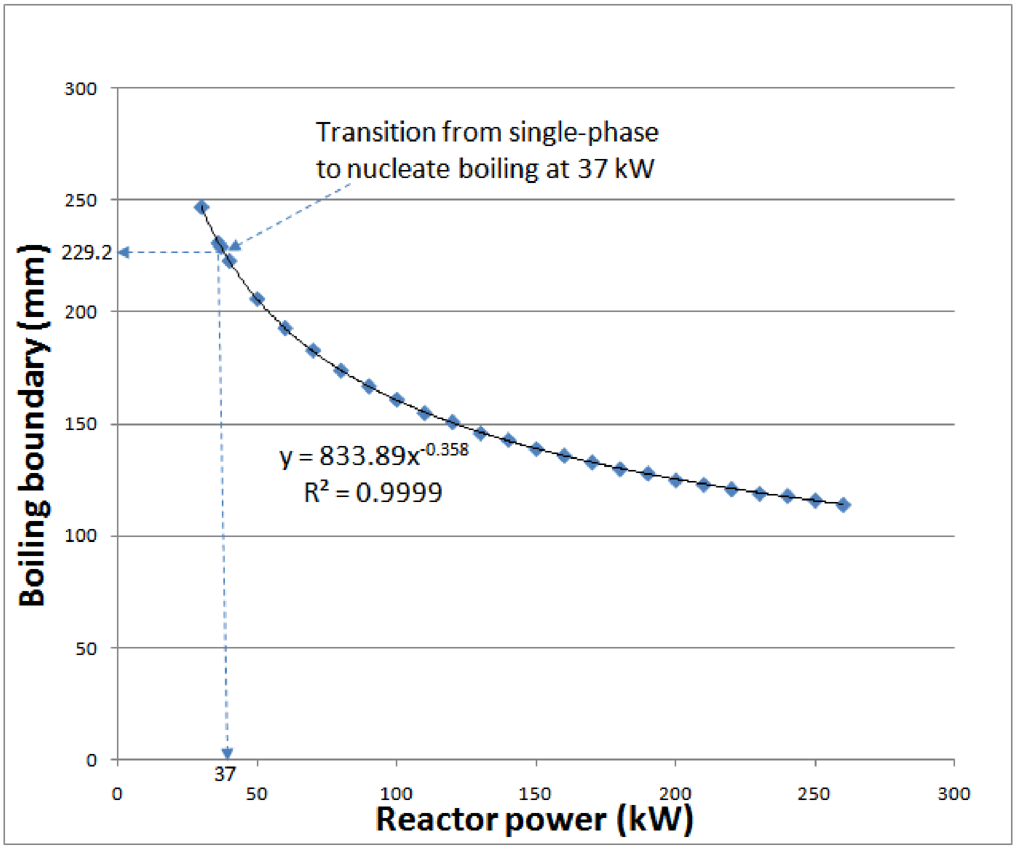

Figure 5: Variation of boiling boundary with reactor power

RESULTS AND DISCUSSIONS

It would be desirable to compare all the calculated values to experimental ones. To my knowledge, there is some existing experimental result to compare the boiling boundary results but there are no existing experimental data to compare the other results. The trends of the results obtained from the present analysis were compared with that of the other research reactors. The results of pressure drop, single-phase heat transfer coefficient and critical heat flux depended on the results of the calculated boiling boundary results as most of the parameters used for calculating them were obtained from the boiling boundary results. The results of the nucleate boiling heat transfer coefficient and onset of nucleate boiling heat flux did not depend on the calculated boiling boundary results. The nucleate boiling heat transfer coefficients values were obtained using Mikheev correlation. The universally acceptable Bergles and Rohsenow correlation was used to obtain the heat flux at the onset of nucleate boiling.

The trends of the results of calculation of the two-phase flow properties are shown in Figures 5 to 10.

Using the law of conservation of one dimensional single-phase energy equation, the boiling boundary L_N obtained for 30 kW and 36 kW reactor powers respectively are 247 mm and 231 mm, which are greater than the active channel height of 230 mm. This indicates that the reactor powers, 30 kW and 36 kW, cannot cause nucleate boiling. The boiling boundary obtained for 37 kW reactor power is 229.2 mm, which is less than the active channel height of 230 mm. This also indicates that the reactor powers above 37 kW can cause nucleation and hence the transition from single-phase to nucleation occurs at 37 kW reactor power. The boiling boundary result of 37 kW agrees with the experimental result of the coolant flow through the 30 kW GHARR-1 core is only single-phase. The result also agrees with the results of the MNSR-TH code (Thermal hydraulics code for analyzing Tank-in-pool Research Reactor Core) developed by Akahoand Dagadu, 1999 which showed that the 36 kW reactor power cannot cause nucleation. The results also show that saturated nucleate boiling is expected at the exit of the core when power excursion is above 240 kW with 118 mm boiling boundary from the inlet. The boiling boundary for the maximum reactor power of 260 kW used in this theoretical research is 114 mm from the inlet. The boiling boundary for experimental abnormal power of 100.2 kW and predicted abnormal powers of 187.23 kW and 254.3 kW are respectively 161 mm, 126 mm and 115 mm. The abnormal reactor powers 187.23 kW and 254.3 kW correspond to inadvertent reactivity insertions of 6.71 mk and 9 mk during the time of installation of a new core and addition of incorrect thickness of top Be plates

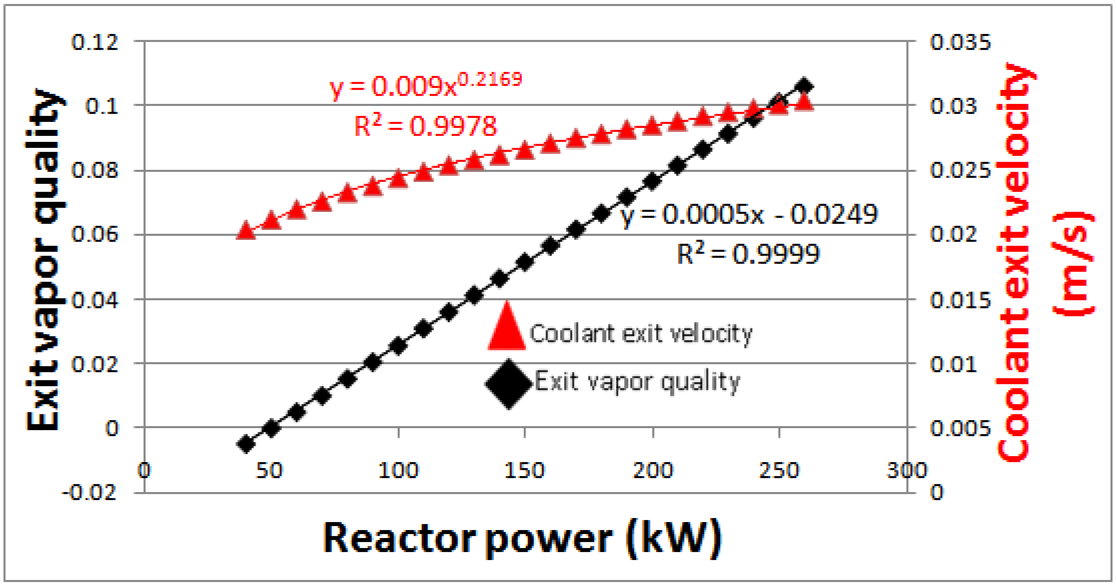

Figure 6: Variation of coolant exit velocity and exit vapor quality with reactor power

respectively. The abnormal reactor power of 100.2 kW corresponds to ramp reactivity insertion of the cold clean core excess reactivity of 4 mk. In general, for fixed inlet coolant pressure of 1 bar (100 kPa), velocity of 10 mm/s and temperature of 30 °C, the boiling boundary from the active core inlet decreases less rapidly with increasing reactor powers as shown in Figure 5.

Figure 6 shows the trend of coolant exit velocity values ranging from 0.0204 m/s to 0.0304 m/s for reactor powers from 40 kW to 260 kW respectively. The values increase less rapidly with increasing reactor powers for the fixed inlet conditions. The coolant exit velocity values of 0.0244 m/s, 0.0281 m/s and 0.0302 m/s were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively. The coolant exit velocities for the various reactor powers were used to estimate the pressure drops across the active height of the channel. The coolant exit velocities were also used to estimate the critical heat flux.

Figure 6 also shows the trend of variation of exit vapor quality with reactor power. The exit vapor quality values range from -0.0048 to 0.1064 for 40 kW to 260 kW reactor powers respectively. Reactor powers with negative vapor quality cannot cause nucleation as there is no generation of vapor. The reactor powers with positive vapor quality can cause nucleation as there is generation of vapor. The exit vapor quality increases linearly with increasing reactor powers for the fixed inlet conditions. The exit vapor quality values of 0.0259, 0.0705 and 0.1036 were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively. From the trend of values, it is not safe for GHARR-1 to be operated at those abnormal powers because of the large fraction of the coolant that could be changed to vapor at the abnormal reactor powers.

The trends of results of coolant exit velocity and exit vapor quality shown in Figure 6 were compared with the trends of the results of steam quality and liquid coolant flow velocity as a function of power level at the core exit of Kuosheng-1 (2894 MWt, 948 MWe) operating at a fixed mass flow rate (Mei-Ya et al., 2008). The trends of exit vapor quality and coolant exit velocity of GHAARR-1 at the abnormal reactor powers and that of Kuosheng-1 reactor are in good agreement.

The conservation law of momentum in one dimension was used to obtain the pressure drop results. In general, for fixed inlet coolant pressure of 1 bar (100 kPa), velocity of 10 mm/s and temperature of 30°C, the pressure drop decreases less rapidly with the increasing reactor powers as shown in Figure 7. In other words, the coolant exit pressure increases less rapidly with increasing reactor powers for those fixed inlet coolant conditions as shown in Figure 7. The drops in the inlet coolant pressure for reactor powers from 40 kW to 260 kW range from 2.203 kPa to 1.153 kPa (97.797 kPa to 98.847 kPa for coolant exit pressures) respectively. The drops in inlet coolant pressure of 1 bar by 1.629 kPa, 1.300 kPa and 1.163 kPa were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively. Considering the inlet coolant pressure of 1 bar (100 kPa), the pressure drops of 2.203 kPa to 1.153 kPa would not have significant effect on the coolant flow through the channel and hence the effective heat removal from the core by the coolant would not be affected.

The values of calculated single-phase heat transfer coefficient *h* range from 1.1612 kW/m²K to 1.7949 kW/m²K for reactor powers from 40 kW to 260 kW respectively. The values increase less rapidly with increasing reactor powers as shown in Figure 8. Values of

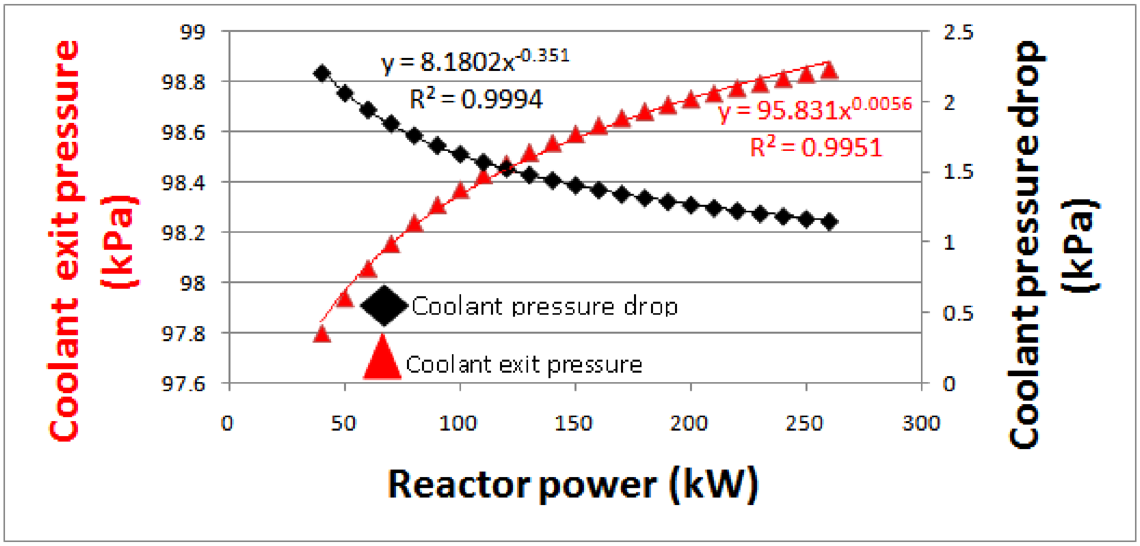

Figure 7: Variation of coolant pressure drop and exit pressure with reactor power

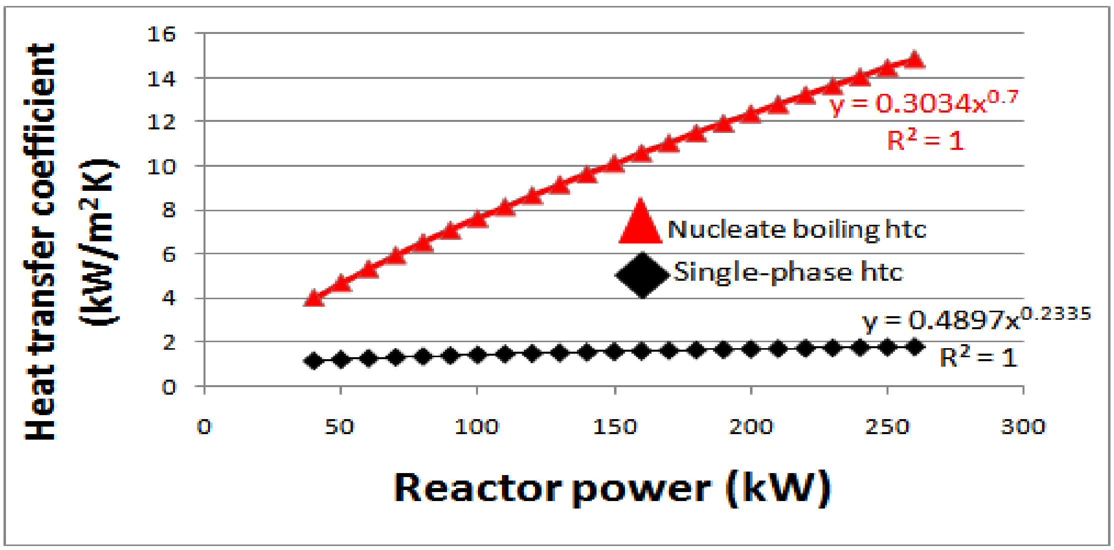

Figure 8: Variation of single-phase and nucleate boiling heat transfer coefficients with reactor power

1.4351 kW/m^{2}K, 1.6614 kW/m^{2}K and 1.7859 kW/m^{2}K were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively. The small values of h indicate low heat transfer from the cladding to the coolant in the single-phase natural convection region and hence effective heat removal by the coolant. The estimation of h was independent of the small coolant velocity, but depended on the physical properties of the coolant, core geometry and temperature difference between the wall and the bulk coolant.

Figure 8 also shows the trend of nucleate boiling heat transfer coefficient h_{nb} values, with the values increasing less rapidly with increasing reactor powers. For reactor powers from 40 kW to 260 kW respectively, the values range from 4.0133 kW/m^{2}K to 14.8780 kW/m^{2}K. Values of 7.6324, 11.9451 and 14.6483 kW/m^{2}K were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively. Comparing the h_{nb} values with h values, the coolant heat removal in the single-phase region will be more effective than in the nucleate boiling region.

The trends of the results of the MNSR single-phase h and nucleate boiling h_{nb} heat transfer coefficients shown in Figure 8 were compared with that of the experimental h and h_{nb} of 250 kW IPR-RI TRIGA Nuclear Research Reactor of Brazil (Mesquita, 2007). The compared trends of h and h_{nb} for GHARR-1 and Brasil TRIGA reactor are in good agreement.

Figure 9 shows the trend of onset of nucleate boiling heat flux q_{onb} values ranging from 11.9491 kW/m^2 to 158.7300 kW/m^2 for reactor powers from 40 kW to 260 kW

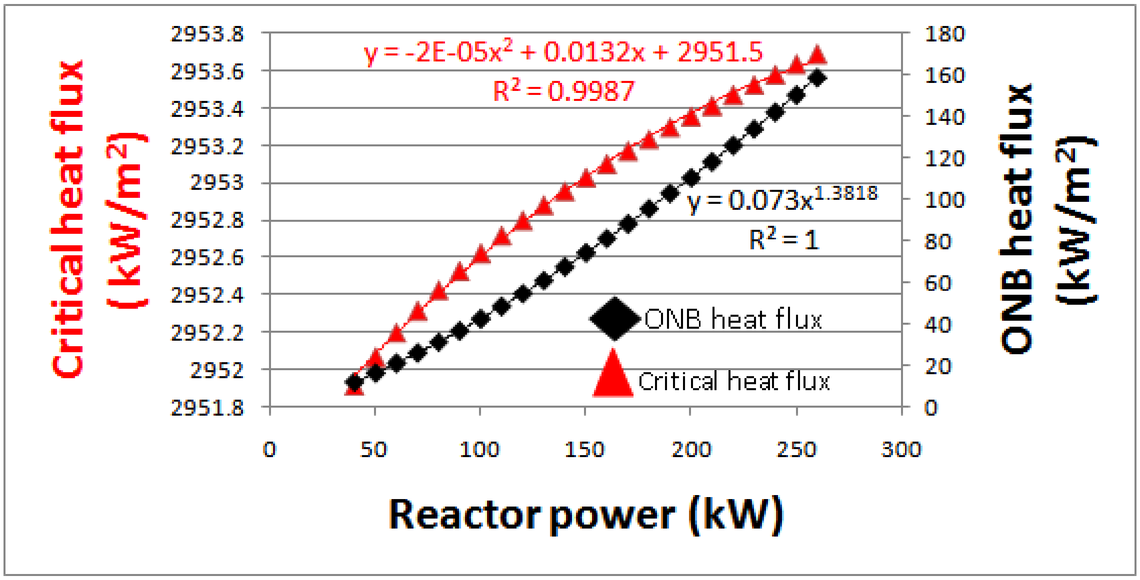

Figure 9: Variation of onset of nucleate boiling heat flux and critical heat flux with reactor power

Table 2. ONBR results of GHARR-1, Pakistan Research Reactor-1 (Pakistan RR-1) and Georgia Technical Research Reactor (Georgia Tech RR).

Reactor	Type of fuel	Power	ONBR
GHARR-1	HEU	30 kW	1.23
Pakistan RR-1	LEU	10 MW	1.4
	Mixed fuel (HEU&LEU)	9.8 MW	1.3
Georgia Tech RR	ANL-HEU	5 MW	1.34
	ANL-LEU	5 MW	1.44

respectively. The values increase less rapidly with increasing reactor powers. The values of 42.5061 kW/m², 100.8501 kW/m² and 153.9574 kW/m² were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively. Comparing the values to actual heat fluxes q_a, which range from 29.2591 kW/m² to 190.1840 kW/m², the q_{onb} values are 40.84 % to 83.46 % of q_a for reactor powers from 40 kW to 260 kW respectively.

The values ranging from 2951.917 kW/m² to 2953.687 kW/m² were the critical heat flux values obtained for reactor powers from 40 kW to 260 kW respectively. The values increase less rapidly with increasing reactor powers as shown in Figure 9. Critical heat flux values of 2952.626, 2953.278 and 2953.657 kW/m² were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively.

The trends of ONB heat flux and critical heat flux values were compared with that of McMaster Nuclear Reactor (Garland et al., 1999). The compared trends of the two research reactors are in good agreement.

The critical heat flux was used to determine the safety margin (DNBR) of the low power reactor. The critical heat flux is a property of the coolant just like the melting point (temperature) of the fuel meat, beyond which heat accumulation in the fuel begins and eventually leading to the melting of the fuel.

The ONBR (the ratio of power at the onset of nucleate boiling to the nominal power) is 1.23 (based on the boiling boundary results). This result shown in Table 2 was compared with the results of Pakistan (Bokhari, 2004) and Georgia Tech (Matos, 1992) Research Reactors. These results show that GHARR-1, Pakistan and Georgia Tech research reactors can only be operated up to at most 140 % of the nominal operating power for the reactor to operate purely in single-phase region.

The Bergles and Rohsenow ONB heat flux correlation was also used to observe the trend of variation of ONBR (the ratio of heat flux at the onset of nucleate boiling to the nominal heat flux) with reactor power as shown in Figure 10. The values increase less rapidly with increasing reactor power. The results showed that ONBR varies from 0.54 to 7.23 for reactor powers from 40 kW to 260 kW respectively. It means that the heat flux that could cause nucleation at 40 kW is 54 % of that of the nominal value and the heat flux that could cause nucleation at 260 kW is 723 % of that of the nominal value. The ONBR values of 1.94, 4.60 and 7.01 were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively.

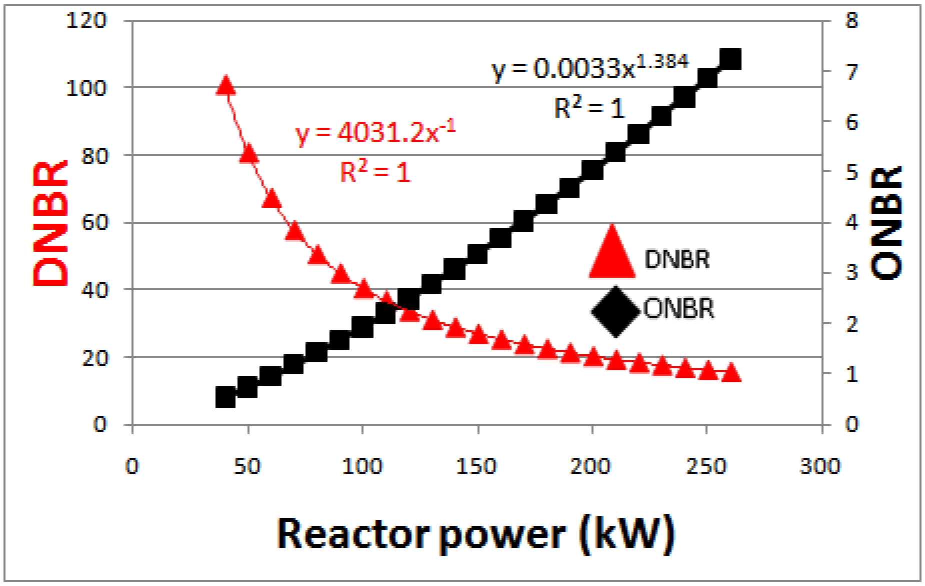

Figure 10: Variation of ONBR and DNBR with reactor power

The operational safety limit of GHARR-1 core should largely be based on the minimum ONBR of 1.23, as this is the limit above which nucleation will start leading to the evaporation of the coolant in the core. It is important to note that the coolant flow through the GHARR-1 core should be single-phase.

Figure 10 also shows the results of DNBR safety margins with the values decreasing less rapidly with increasing reactor powers. The values vary from 100.89 to 15.53 for reactor powers of 40 kW to 260 kW respectively. At 30 kW nominal reactor power, the DNBR is 134.44, which means that during the time of unusual power excursion of 13444 % of the nominal value of 30 kW the surface heat fluxes could attain the critical heat flux values. At 260 kW reactor power with 15.53 DNBR, unusual power excursion of 1553 % of 260 kW could cause the surface heat fluxes to attain the critical heat flux values. The DNBR values of 40.30, 21.58 and 15.88 were obtained for abnormal reactor powers of 100.2 kW, 187.23 kW and 254.3 kW respectively. It is expected that the GHARR-1 core will melt when the power excursion could reach 4000 kW (obtained by multiplying the values of DNBR and its actual operating reactor power) reactor power with 1.008 DNBR. The values show that during the time of unusual power excursion the surface heat fluxes can easily get to the critical heat flux values when the reactor is operating at higher reactor powers as indicated by the low DNBR safety margins. The surface heat fluxes can hardly attain the critical heat flux values at low reactor powers as indicated by the large DNBR safety margins during the time of unusual power excursion. The criteria of thermal-hydraulic design for GHARR-1 MNSR require that the DNBR should be greater than 2.55 (Akaho et al., 2003). Observing the DNBR safety margins, the GHARR-1 MNSR can hardly experience nuclear accident even if it is operating at the abnormal reactor powers. Usually DNBR is an important parameter for water-cooled reactors and represents an operational safety limit. For PWR systems, a minimum value of 1.3 is usually specified as the surface heat fluxes could attain the critical heat flux values with small nucleation at the exit of the reactor core (that is surface heat fluxes could attain critical heat flux values with the coolant temperature less than its saturation temperature) (Zafar, 2006).

CONCLUSIONS

Ghana Research Reactor-1 (GHARR-1) is a low power reactor of 30 kW (th) tank-in-pool type with parallel multichannel core, operating presently with a Highly Enriched Uranium (HEU). The data on GHARR-1 was used for this research work.The GHARR-1 under normal operating conditions experiences single-phase flow. Under abnormal operating conditions GHARR-1 experiences two-phase flow regime which is caused by large reactivity insertions associated with the abnormal operating conditions of the low power reactor.This theoretical research purposely investigated two-phase flow properties (parameters) which are normally associated with nucleate boiling that develops at abnormal operating conditions of the low power reactor. A FORTRAN code, conservation laws and correlations were employed to compute the two-phase flow parameters, which include boiling boundary, vapor quality, pressure drop, single-phase and nucleate boiling heat transfer coefficients, onset of nucleate boiling (ONB) heat flux, critical heat flux and safety margins (ONB ratio (ONBR) and DNB ratio (DNBR)). The following conclusions were obtained as a results of the study.

1. Subcooled nucleate boiling will commence if 30 kW GHARR-1 MNSR is operated above 37 kW reactor power. This was indicated by its boiling boundary of 229.2 mm from the inlet at 37 kW reactor power and ONBR of 1.23. Saturated nucleate boiling is expected at the exit of the core when power excursion is above 240 kW with 118 mm boiling boundary from the inlet.
2. The drops in inlet coolant pressure of 1 bar (100 kPa) will have negligible effects on the effective heat removal by the coolant in the core. This was indicated by pressure drops of 2.203-1.153 kPa for reactor powers from 40 kW to 260 kW respectively.
3. The coolant in the GHARR-1 core will not evaporate if it is operated below 40 kW reactor power. This was confirmed by its exit vapor quality of -0.0048 at 40 kW reactor power.
4. The coolant heat removal from the core at low reactor powers will be more effective than its heat removal at high reactor powers. This was indicated by low single-phase heat transfer coefficient values (1.1612-1.7949 kW/m^2K) compared to high nucleate boiling heat transfer coefficient values(4.0133-14.8780 kW/m^2K) for 40 kW to 260 kW reactor powers respectively.
5. Low power reactors operating at low reactor powers can hardly experience nuclear accident. This was confirmed by low power GHARR-1 MNSR large DNBR of 100.89 to 15.53 for 40 kW to 260 kW reactor powers respectively. The DNBR of 30 kW GHARR-1 MNSR is 134.44. It is expected that the GHARR-1 core will melt when the power excursion could reach 4000 kW reactor power with 1.008 DNBR.
6. Low power GHARR-1 MNSR is safe during the time of abnormal operations and can have longer life span if it is continued to be operated at low reactor powers.

Acknowledgements

The authors acknowledged financial support provided by Ghana Atomic Energy Commissionfor this work.

REFERENCES

Abdulhameed S, Balogun GI(2017). Prediction of Peak Temperatures of Nigeria Research Reactor Core Components under Several Reactivity Accident Tests. FUOYE J. Engineer. Technol., 2 (1): 67-71.

Adiutori EF (2004). An Improved Form for Natural Convection Heat Transfer Correlations. Proceedings of the ASME-ZSIS International Thermal Science Seminar II, Bled, Slovenia.

Ahmed A, Balogun GI, Jonah SA, Funtua II (2008). The behavior of reactor power and flux resulting from changes in core coolant temperature for a miniature neutron source reactor. Annals of Nuclear Energy 35: 2417-2419.

Akaho EHK, Anim-Sampong S, Dodoo-Amoo DNA, Maakuu BT, Emi-Reynolds G, Osae EK, Boadu HO, Bamford SA(2003). Ghana Research Reactor-1 Final Safety Analysis Report. Ghana Atomic Energy Commission Technical Report, GAEC-NNRI-RT-90.

Akaho EHK, Dagadu CPK (1999). MNSR-TH: ThermaL-hydraulics code for analyzing Tank-in-Pool Type Research Reactor Core. Proceedings of the First AFRA Regional Conference in Research Reactor Operation, Safety and Utilization, vol. 2, Algiers, Algeria.

Akaho EHK, Maaku BT(2002). Simulation of reactivity transients in a miniature neutron source reactor core. Nuclear Engineering and Design, Volume 213: 31-42.

Ampomah-Amoako E, Akaho EHK, Sampong SA, Nyarko BJB(2009). Transient analysis of Ghana Research Reactor-1 using PARET/ANL thermal- hydraulic code, Nuclear Engineering and Design, 239: 2479-2483.

Birikorang SA Akaho EHK, Nyarko BJB(2010). Comparative assessment of theoretical and experimental measurement of photon neutron flux level in GHARR- 1 after shutdown. Nuclear Engineering and Design, 240: 2731-2736.

Bokhari IH (2004). Steady-State Thermal Hydraulic and Safety Analysis of a proposed mixed fuel (HEU & LEU) core for Pakistan Research Reactor-1. Annals of Nuclear Energy, 31: 1265-1273.

El-Sawy TH, Ashoub N, Fathallah A(2001). Power Upgrading Study for the First Egyptian Research Reactor. Nuclear Engineering and Design, 208: 245-254.

Garland WJ, Butler M, Saunders F (1999). Heat Transfer Limits for the McMaster Nuclear Reactor. Technical Report 1999-01: 1-15.

Guo Y, Qiu SZ, Su GH, Jia DN(2008). Theoretical investigations on two-phase flow instability in parallel multichannel system. Annals of Nuclear Energy, 35: 665-676.

Guo Y, Su GH, Wang JQ, Tian WX, Qiu SZ, Jia DN, Zhang JW(2005). Two-phase instability analysis in natural circulation loops of China advanced research reactor. Annals of Nuclear Energy, 32: 379-397.

Mangena S (2017). Thermal Hydraulic and Safety Analysis of Heat Transfer and Distribution in the Ghana Research Reactor-1 (GHARR-1) core using STAR-CCM+ CFD Code. MPhil Thesis submitted to the Graduate School of Nuclear and Allied Sciences, University of Ghana.

Matos JE, Mo SC, Woodruff WL (1992). Analyses for Conversion of the Georgia Tech Research Reactor from HEU to LEU fuel. RERTR Programme, Argonne National Laboratory, Argonne, IL 60439: 1-42.

Mei-Ya W, Tsung-Kuang Y, Chu CF (2008). Predicted impact of power uprate on the water chemistry of Kuosheng boiling water reactor. Nuclear Engineering and Design, 238: 2746-2753.

Mesquita AZ(2007). Experimental Heat Transfer Analysis of the IPR-RI TRIGA Reactor of Nuclear Technology Development Centre (CDTN), Belo Horizonte, Brazil.

Mweetwa BM, Ampomah-Amoako E, Akaho EHK(2017). Transient Studies of Ghana Research Reactor-1 after

Nineteen (19) Years of Operation Using PARET/ANL Code. World Journal of Nuclear Science and Technology 7, 223-231.

Tanaka F, Hibiki T, Saito Y, Takeda T, Mishima K (2001). Heat Transfer Study for Thermal-Hydraulic Design of the solid-Target of Spallation Neutron Source. J. Nuclear Sci. Technol., 38 (10): 832-843.

Zafar UK(2006). An Elementary Model to Study Sensitivity of the Departure from Nucleate Boiling Ratio (DNBR) to Perturbations in Nuclear Reactor Systems. ARPN Journal of Engineering and Applied Sciences, Vol.1, No.1.

APPENDIX

Calculation of boiling boundary

Conservation of one dimensional single-phase energy equation (Guo et al., 2008) is given by

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial z}(\rho h u) = q''' \qquad (A1)$$

Based on the assumptions that the coolant density and velocity were constant, the energy equation becomes

$$\rho\frac{dh}{dt} + \rho u\frac{dh}{dz} = q''' \qquad (A2)$$

The energy equation was integrated over one node with boundaries, L_n and L_{n-1}, to obtain

$$\rho h\frac{d}{dt}(L_n - L_{n-1}) + \rho h u = q'''(L_n - L_{n-1}) \qquad (A3)$$

$$\frac{dL_n}{dt} = \frac{dL_{n-1}}{dt} - u_i + \frac{q'''}{\rho h}(L_n - L_{n-1}) \qquad (A4)$$

$$L_n = L_{n-1} - u_i t + qt(L_n - L_{n-1}), \qquad q = \frac{q'''}{\rho h} \qquad (A5)$$

$$L_n = \frac{tu_i + L_{n-1}(qt-1)}{qt-1} \qquad (A6)$$

If n = 1

$$L_1 = \frac{tu_i}{qt-1}, \qquad q = \frac{q'''}{\rho h}, q''' = \frac{p}{V_{cv}} \quad (eqn\ 3.8) \qquad (A7)$$

$$L_1 = \frac{tu_i\rho h V_{cv}}{pt - V_{cv}\rho h} \qquad (A8)$$

$$L_N = NL_1 = \frac{Ntu_i\rho h V_{cv}}{pt - \rho h V_{cv}} \qquad (A9)$$

where

u_i (u) is the coolant inlet velocity (m/s), h is enthalpy (J/kg) for one node, q''' is power density (W/m^3), energy released per unit volume of the active reactor core, L_N is the boiling boundary (m) from the inlet of the active core, p is power (W), t is time (s), ρ is coolant density (kg/m^3), V_{cv} is the volume of the active reactor core and N is the cell (node) number of the single-phase.

Fluid properties were evaluated at an average temperature of $\left(\frac{T_i+T_e}{2}\right)$.

The enthalpy for one node is given by

$$h = h_n - h_{n-1} = \frac{h_e - h_i}{N} \qquad (A10)$$

Subscripts i and e denote the inlet and exit of the heater section.

The boiling boundary L_N was used to find the fluid boiling boundary enthalpy h_N by assuming that the enthalpy variation of the bulk fluid through the heated channel is linear. The coolant boiling boundary enthalpy $h_N$at any reactor power is given by

$$h_N = \frac{L_N}{L_H}(h_e - h_i) + h_i \qquad (A11)$$

whereh_e is the coolant exit enthalpy at the exit of the heater section for any reactor power and L_H is the length of the heater section.

The coolant boiling boundary enthalpy h_N is greater than the coolant exit enthalpy h_e if the boiling boundary L_N is greater than the heater section length L_H. This means that the reactor operating power cannot cause nucleate boiling.

Calculation of exit velocity of the coolant

Coolant exit velocity at the exit of the heater section is needed for the estimation of pressure drop across the channel and also for the estimation of critical heat flux.

Calculation of the coolant exit velocity was done by combining the conservation of one dimensional mass and energy equations (Guo et al., 2008).

The one dimensional conservation of mass equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho u) = 0 \qquad (A12)$$

The one dimensional conservation of energy equation is also given by

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial z}(\rho h u) = q''' \qquad (A13)$$

From equation (A13)

$$\rho \frac{\partial h}{\partial t} + h \frac{\partial \rho}{\partial t} + \rho h \frac{\partial u}{\partial z} + h u \frac{\partial \rho}{\partial z} + \rho u \frac{\partial h}{\partial z} = q''' \qquad (A14)$$

From equation (A12)

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial z} + u \frac{\partial \rho}{\partial z} = 0 \qquad (A15)$$

$$\frac{\partial \rho}{\partial t} = - \rho \frac{\partial u}{\partial z} - u \frac{\partial \rho}{\partial z} \qquad (A16)$$

Putting equation (A16) into (A14) gives

$$\rho \frac{\partial h}{\partial t} - \rho h \frac{\partial u}{\partial z} - h u \frac{\partial \rho}{\partial z} + \rho h \frac{\partial u}{\partial z} + h u \frac{\partial \rho}{\partial z} + \rho u \frac{\partial h}{\partial z} = q''' \qquad (A17)$$

$$\rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial z} = q''' \qquad (A18)$$

Integrating (A18) with respect to time gives

$$\rho h + t \rho h \frac{du}{dz} = q''' t \qquad (A19)$$

$$\frac{du}{dz} = \frac{q''' t - \rho h}{t \rho h} \qquad (A20)$$

Integrating the resulting velocity equation from the boiling boundary to the exit of the reactor core (heater section) gives

$$u_e = u_i + \frac{q''' t - \rho h}{t \rho h} \Delta L_{SBL} \, , q''' = \frac{p}{V_{cv}} \qquad (A21)$$

$$u_e = u_i + \frac{p t - V_{cv} \rho h}{V_{cv} t \rho h} \Delta L_{SBL} \qquad (A22)$$

where u_i and u_e are the inlet (boiling boundary) and exit coolant velocities respectively, ρ is the coolant density in the nucleate boiling region, p is power, t is time, ΔL_{SBL} is subcooled boiling length and h is subcooled boiling enthalpy given by

$$h = h_e - h_N \qquad (A23)$$